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Heat Transfer from Rotating Porous Plate Using Homotopy Perturbation Method

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The homotopy perturbation method is employed to solve the nonlinear energy equation due to temperature-dependent thermal conductivity. The effects of variable thermal conductivity and other parameters on heat transfer are investigated for a hydromagnetic flow of an incompressible viscous electrically conducting fluid past a rotating porous plate. The plate is rotating with a uniform angular velocity about an axis normal to the plate, and the fluid is rotating at infinity with the same angular velocity about a noncoincident parallel axis. It is demonstrated that the dimensionless heat transfer decreases along the vertical distance from the plate surface with the decrease in thermal conductivity and increases with the increase in suction parameter, Brinkman number, magnetic parameter, or Prandtl numbers, while the others are kept constant.

Nomenclature

B	=	magnetic field
Br	=	Brinkman number
b	=	temperature coefficient of thermal conductivity
c_p	=	specific heat of fluid, $\mathrm{kJ/kg \cdot K}$
Ec	=	Eckert number
J	=	current density
k	=	thermal conductivity, $\mathrm{W/m \cdot K}$
ℓ	=	distance between two axes, m
M	=	dimensionless magnetic parameter
Pr	=	Prandtl number
p	=	embedding parameter
q^*	=	dimensionless heat transfer
S	=	dimensionless suction parameter
T	=	temperature, °C
w_0	=	suction/blowing velocity, m/s
u, v	=	velocity components in x and y directions, m/s
α, β	=	constants defined in Eqs. (14) and (15) based on suction
α_1, β_1	=	constants defined in Eqs. (17) and (18) based on blowing
ζ	=	dimensionless distance from plate
θ	=	dimensionless temperature
μ	=	absolute viscosity of fluid, $\mathrm{kg/m \cdot s}$
ν	=	kinematic viscosity of fluid, $\mathrm{m^2/s}$
ρ	=	fluid thermal conductivity, $\mathrm{W/m \cdot K}$
σ	=	fluid density, $\mathrm{kg/m^3}$
Ω	=	angular velocity, rad/s

Subscripts

w	=	wall
0	=	initial guess
∞	=	freestream condition

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I. Introduction

MAGNETOHYDRODYNAMIC (MHD) boundary layers under the influence of viscous forces are of great importance in understanding a variety of geophysical, astrophysical, and engineering phenomena, such as those that occur at the core–mantle interface of the earth. The flow of an incompressible viscous fluid between two parallel plates rotating noncoaxially but with the same angular velocity was first studied by Berker [1]. Later on, Coirier [2], Erdogan [3–5], Pop [6], Rajagopal [7], Kasiviswanathan and Ramachandra [8], Erosy [9], and Hayat et al. [10–12] studied the flow due to a disk and a fluid at infinity that were rotating noncoaxially at a slightly different angular velocity.

Millsaps and Pohlhausen [13], Sparrow and Gregg [14,15], Tadros and Erian [16], Evans and Greif [17], Palec [18], Hirose et al. [19], and Attia [20] determined heat transfer from a rotating disk for various thermal conditions in the steady state. Recently, Chakraborti et al. [21] solved the same problem and determined a solution for heat transfer with constant thermal conductivity. In all these studies, constant thermal conductivity was considered. The main objective of this paper is to investigate the effects of temperature-dependent thermal conductivity on heat transfer for an electrically conducting incompressible viscous fluid past a porous plate in the presence of a uniform transverse magnetic field. Analytical expressions for the temperature profile and the heat transfer from the plate with Dirichlet condition on the surface are determined using the homotopy perturbation method (HPM) given by He [22].

II. Basic Idea of Homotopy Perturbation Method

Consider a differential equation,

$$A(u) - f(r) = 0; \qquad r \in \mathbb{R} \tag{1}$$

with boundary conditions,

$$B'\left(u, \frac{\partial u}{\partial \eta}\right) = 0; \qquad r \in \partial\mathbb{R} \tag{2}$$

where A is a general nonlinear operator. B' is a boundary operator, $f(r)$ is known as an analytic function, $\partial\mathbb{R}$ is the boundary domain, and $\frac{\partial u}{\partial \eta}$ is the directional derivative. The nonlinear operator A can be divided further into linear L and nonlinear N parts, so that Eq. (1) can be expressed as

$$L(u) + N(u) - f(r) = 0 \tag{3}$$

where $f(r)$ is an analytic function in the given equation. The homotopy of Eq. (3) can be constructed as follows:

$$v(r, p)\colon \mathbb{R} \times [0, 1] \to R \tag{4}$$

which satisfies the equation

$$\mathcal{H}[v, p] = (1 - p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0 \tag{5}$$

where $p \in [0, 1]$ is an embedding parameter, and u_0 is the initial guess. Therefore, Eq. (5) can be expressed as

$$\mathcal{H}[v, p] = L(v) - L(u_0) + p[L(u_0) + N(v) - f(r)] \tag{6}$$

with conditions

$$v(r, 0) = u_0; \qquad v(r, 1) = u(r) \tag{7}$$

The solution of Eq. (6) with Eq. (7) can be expressed as follows:

$$v(r, p) = \sum_{n=1} p^{n-1} v_{n-1} \tag{8}$$

III. Analysis

Consider a porous plate coincident with the plane $z = 0$ and rotating about the z axis, with uniform angular velocity Ω, in an incompressible viscous electrically conducting fluid with thermal conductivity σ. The plate is assumed to be electrically nonconducting. The geometry of the problem is shown in Fig. 1. A uniform magnetic field B is applied parallel to the z axis, and the fluid is rotating about an axis parallel to the z axis with the same angular velocity Ω. The plate is maintained at a constant temperature T_w. The distance between both axes of rotation is ℓ.

Let (u, v) be the velocity components in the x and y directions, respectively. Following Chakraborti et al. [21], the hydrodynamic boundary conditions can be written as

$$\begin{aligned} &z = 0\colon\ u = -\Omega y; \qquad v = \Omega x; \qquad w = -w_0; \\ &z \to \infty\colon\ u \to -\Omega(y - l); \qquad v \to \Omega x; \qquad w = 0 \end{aligned} \tag{9}$$

where w_0 is the suction/blowing velocity at the plate. Chakraborti et al. [21] suggested the following velocity field for the plate:

$$u = -\Omega y + f(z); \qquad v = \Omega x + g(z) \tag{10}$$

where $f(z)$ and $g(z)$ are the components of the velocity field in the direction normal to the plane containing the axis of rotation and in the transverse direction parallel to the plane of the plate, respectively. Chakraborti et al. [21] used the following equations of momentum along the x and y directions:

$$\begin{aligned} &v f_1'' + w_0 f_1' - \frac{\sigma B^2}{\rho} f_1 = \frac{1}{\rho}\frac{\partial p}{\partial x} - \Omega^2 x - \Omega g; \\ &v g'' + w_0 g' - \frac{\sigma B^2}{\rho} g = \frac{1}{\rho}\frac{\partial p}{\partial y} - \Omega^2 y + \Omega f_1 \end{aligned} \tag{11}$$

where the prime denotes differentiation with respect to z. Using mass and momentum equations, they obtained these components in dimensionless form for suction:

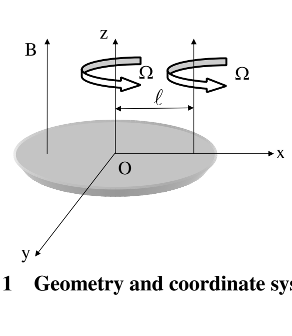

Fig. 1 Geometry and coordinate system.

$$\frac{f_1(z)}{\Omega l} = 1 - e^{(-\zeta\alpha)}\cos(\zeta\beta) \tag{12}$$

$$\frac{g(z)}{\Omega l} = e^{(-\zeta\alpha)}\sin(\beta\zeta) \tag{13}$$

where

$$\zeta = \left(\frac{\Omega}{2v}\right)^{1/2} z \tag{14}$$

$$\alpha = S + \left[\sqrt{1 + \frac{(S^2 + M)^2}{4}} + \frac{(S^2 + M)}{2}\right]^{1/2} \tag{15}$$

$$\beta = \left[\sqrt{1 + \frac{(S^2 + M)^2}{4}} - \frac{(S^2 + M)}{2}\right]^{1/2} \tag{16}$$

with the dimensionless suction parameter S and the magnetic parameter M, respectively, as

$$S = \frac{w_0}{(2v\Omega)^{1/2}}; \qquad M = \frac{2\sigma B^2}{\rho\Omega} \tag{17}$$

For blowing at the plate ($S < 0$), the dimensionless velocity components are the same as given by Eqs. (10). In this case, we replace S with $S_1 > 0$ in Eqs. (15) and (16), so that, for blowing at the plate,

$$\alpha_1 = S_1 + \left[\sqrt{1 + \frac{(S_1^2 + M)^2}{4}} + \frac{(S_1^2 + M)}{2}\right]^{1/2} \tag{18}$$

$$\beta_1 = \left[\sqrt{1 + \frac{(S_1^2 + M)^2}{4}} - \frac{(S_1^2 + M)}{2}\right]^{1/2} \tag{19}$$

IV. Heat Transfer

The governing boundary-layer equation with viscous dissipation and joule effect for the steady-state condition can be written as [21]

$$\begin{aligned} -\rho c_p w_0 \frac{\mathrm{d}T}{\mathrm{d}z} &= \frac{\mathrm{d}}{\mathrm{d}z}\left[k(T)\frac{\mathrm{d}T}{\mathrm{d}z}\right] + \mu\left[\left(\frac{\mathrm{d}u}{\mathrm{d}z}\right)^2 + \left(\frac{\mathrm{d}v}{\mathrm{d}z}\right)^2\right] \\ &\quad + \frac{1}{\sigma}[J_x^2 + J_y^2] \end{aligned} \tag{20}$$

where c_p is the specific heat of the fluid, and $k(T)$ is the thermal conductivity of the material that varies with temperature. When this variation in the range of practical interest is large, it is necessary to account for this variation to minimize the error in heat transfer. Accounting for the variation of the thermal conductivity with temperature makes the governing conduction equation nonlinear. The variation in thermal conductivity of a material with the temperature can be approximated in the following manner:

$$k(T) = k_a[1 + b(T - T_\infty)] \tag{21}$$

where k_a is the thermal conductivity of the material at the reference temperature, and b is the temperature coefficient of thermal conductivity. This temperature coefficient may be positive or negative, depending upon heating or cooling. The thermal boundary conditions are

$$T = T_w \quad \text{at } z = 0 \tag{22}$$

and

$$T \to T_\infty \quad \text{as } z \to \infty$$

T_w and T_∞ are constants, with $T_w > T_\infty$.

Introducing the following dimensionless parameters in Eq. (11),

$$\left.\begin{array}{c} \theta(\zeta) = \frac{T(z)-T_\infty}{T_w-T_\infty} \\ \zeta = \left(\frac{\Omega}{2\nu}\right)^{1/2} z \\ \epsilon = b(T_w - T_\infty) \\ Pr = \frac{\mu c_p}{k_a} \\ Ec = \frac{\Omega^2 l^2}{c_p(T_w-T_\infty)} \\ Br = PrEc \end{array}\right\} \tag{23}$$

and simplifying, we get the nonlinear energy equation,

$$\frac{d^2\theta}{d\zeta^2} + \epsilon\theta\frac{d^2\theta}{d\zeta^2} + \epsilon\left(\frac{d\theta}{d\zeta}\right)^2 + (2SPr)\frac{d\theta}{d\zeta} + Bre^{-2\alpha\zeta}(\alpha^2 + \beta^2 + M) = 0 \tag{24}$$

with the dimensionless thermal boundary conditions:

$$\theta(0) = 1 \quad \text{and} \quad \theta(\infty) = 0 \tag{25}$$

V. Solution of the Problem

According to the HPM, Eq. (23) is decomposed into a linear part $L(\theta)$, a nonlinear part $N(\theta)$, and a source term $f(r)$; that is,

$$\left.\begin{array}{c} L(\theta) = \frac{d^2\theta}{d\zeta^2} + (2SPr)\frac{d\theta}{d\zeta} \\ N(\theta) = \epsilon\left[\theta\frac{d^2\theta}{d\zeta^2} + \left(\frac{d\theta}{d\zeta}\right)^2\right] \\ f(\zeta) = -Bre^{-2\alpha\zeta}(\alpha^2 + \beta^2 + M) \end{array}\right\} \tag{26}$$

Now, we construct a homotopy $\theta(x, p)$: $\Omega \times [0, 1] \to R$ that satisfies the equation

$$(1 - p)[L(\theta) - L(\theta_0)] + p[N(\theta) - f(\zeta)] = 0 \tag{27}$$

where $p \in [0, 1]$ is an embedding parameter, and θ_0 is the initial solution that can be obtained by using

$$\frac{d^2\theta_0}{d\zeta^2} + (2SPr)\frac{d\theta_0}{d\zeta} + Br(\alpha^2 + \beta^2 + M)e^{-2\alpha\zeta} = 0 \tag{28}$$

Using the boundary conditions

$$\theta_0(0) = 1 \quad \text{and} \quad \theta_0(\infty) = 0 \tag{29}$$

The solution of Eq. (27) using conditions in Eq. (28) can be written as

$$\theta_0(\zeta) = e^{-(2SPr)\zeta} + \frac{Br(\alpha^2 + \beta^2 + M)}{4\alpha(SPr - \alpha)}[e^{-(2\alpha)\zeta} - e^{-(2SPr)\zeta}] \quad (\text{when } SPr \neq a) \tag{30}$$

which is the same as obtained by Chakraborti et al. [21] for constant thermal conductivity.

We assume that Eq. (23) has a solution of the form

$$\theta(\zeta, p) = \theta_0 + p\theta_1 + p^2\theta_2 + \cdots \tag{31}$$

By substituting Eq. (30) into Eq. (23) and equating the powers of p, we obtain

$$\begin{aligned} \frac{d^2\theta_1}{d\zeta^2} + 2SPr\frac{d\theta_1}{d\zeta} = &-\frac{\alpha A}{(SPr - \alpha)}e^{-2\alpha\zeta} + \frac{SPrA}{(SPr - \alpha)}e^{-2\alpha\zeta} \\ &- \epsilon\Bigg\{8S^2Pr^2e^{-4SPr\zeta} + \frac{\alpha A}{(SPr - \alpha)}e^{-2\zeta(SPr+\alpha)} \\ &- \frac{4S^2Pr^2A}{\alpha(SPr - \alpha)}e^{-4SPr\zeta} + \frac{S^2Pr^2A}{\alpha(SPr - \alpha)}e^{-2\zeta(SPr+\alpha)} \\ &+ \frac{\alpha A^2}{4\alpha(SPr - \alpha)^2}e^{-4\alpha\zeta} - \frac{S^2Pr^2A^2}{4\alpha^2(SPr - \alpha)^2}e^{-2\zeta(SPr+\alpha)} \\ &- \frac{\alpha A^2}{4\alpha(SPr - \alpha)}e^{-2\zeta(SPr+\alpha)} + \frac{S^2Pr^2A^2}{2\alpha^2(SPr - \alpha)^2}e^{-4SPr\zeta}\Bigg\} \\ &+ \frac{A^2}{4(SPr - \alpha)^2}e^{-4\alpha\zeta} + \frac{2SPrA}{(SPr - \alpha)}e^{-2\zeta(SPr+\alpha)} \\ &- \frac{SPrA^2}{2\alpha(SPr - \alpha)^2}e^{-2\zeta(SPr+\alpha)} + e^{-2\alpha\zeta}A \end{aligned} \tag{32}$$

where $A = Br(\alpha^2 + \beta^2 + M)$, and the Dirichlet conditions are

$$\theta_1(0) = 0 \quad \text{and} \quad \theta_1(\infty) = 0 \tag{33}$$

The homogeneous and particular solutions of Eq. (31) can be written as

$$\theta_{1c}(\zeta) = C_3 + C_4e^{-2SPr\zeta} \tag{34}$$

$$\begin{aligned} \theta_{1p}(\zeta) = &\frac{\alpha A}{4\alpha(SPr-\alpha)^2}e^{-2\alpha\zeta} - \frac{SPrA}{4\alpha(SPr-\alpha)}e^{-2\alpha\zeta} \\ &- \epsilon\Bigg\{e^{-4SPr\zeta} + \frac{\alpha A}{4\alpha(S^2Pr^2-\alpha^2)}e^{-2\zeta(SPr+\alpha)} \\ &- \frac{A}{2\alpha(SPr-\alpha)}e^{-4SPr\zeta} + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)}e^{-2\zeta(SPr+\alpha)} \\ &- \frac{A^2}{32\alpha(SPr-\alpha)^3}e^{-4\alpha\zeta} - \frac{S^2Pr^2A^2}{164\alpha^3(SPr-\alpha)^2(SPr+\alpha)}e^{-2\zeta(SPr+\alpha)}\Bigg\} \\ &- \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)}e^{-2\zeta(SPr+\alpha)} \\ &+ \frac{A^2}{16\alpha^2(SPr-\alpha)^2}e^{-4SPr\zeta} - \frac{A^2}{32(SPr-\alpha)^3}e^{-4\alpha\zeta} \end{aligned} \tag{35}$$

Using the boundary conditions (32), we get

$$\begin{aligned} &C_3 = 0 \quad \text{and} \\ &C_4 = -\frac{\alpha A}{4\alpha(SPr - \alpha)^2} + \frac{SPrA}{4\alpha(SPr - \alpha)} \\ &\quad + \epsilon\Bigg[1 + \frac{\alpha A}{4\alpha(S^2Pr^2 - \alpha^2)} - \frac{A}{2\alpha(SPr - \alpha)} \\ &\quad + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2 - \alpha^2)} - \frac{A^2}{32\alpha(SPr - \alpha)^3} \\ &\quad - \frac{S^2Pr^2A^2}{164\alpha^3(SPr - \alpha)^2(SPr + \alpha)}\Bigg] - \frac{A^2}{16\alpha(SPr - \alpha)(SPr + \alpha)} \\ &\quad + \frac{A^2}{16\alpha^2(SPr - \alpha)^2} - \frac{A^2}{32(SPr - \alpha)^3} \\ &\quad + \frac{2SPrA}{4\alpha(SPr - \alpha)(SPr + \alpha)} - \frac{SPrA^2}{8\alpha^2(SPr - \alpha)^2(SPr + \alpha)} \\ &\quad + \frac{A}{4\alpha(SPr - \alpha)} \end{aligned}$$

Substituting the values of C_3 and C_4 in Eq. (33), we get

$$
\begin{aligned}
\theta_1(\zeta) = \theta_{1c} + \theta_{1p} = \Bigg[& -\frac{\alpha A}{4\alpha(SPr-\alpha)^2} + \frac{SPrA}{4\alpha(SPr-\alpha)} \\
& + \epsilon\Bigg\{ 1 + \frac{\alpha A}{4\alpha(S^2Pr^2-\alpha^2)} - \frac{A}{2\alpha(SPr-\alpha)} \\
& + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} - \frac{A^2}{32\alpha(SPr-\alpha)^3} \\
& - \frac{S^2Pr^2A^2}{164\alpha^3(SPr-\alpha)^2(SPr+\alpha)} \Bigg\} - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} \\
& + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} - \frac{A^2}{32(SPr-\alpha)^3} \\
& + \frac{2SPrA}{4\alpha(SPr-\alpha)(SPr+\alpha)} - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} \\
& + \frac{A}{4\alpha(SPr-\alpha)} \Bigg] e^{-2SPr\zeta} + \frac{\alpha A}{4\alpha(SPr-\alpha)^2} e^{-2\alpha\zeta} \\
& - \frac{SPrA}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} - \epsilon\Bigg[e^{-4SPr\zeta} + \frac{\alpha A}{4\alpha(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{A}{2\alpha(SPr-\alpha)} e^{-4SPr\zeta} + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} \Bigg] \\
& - \frac{A^2}{32\alpha(SPr-\alpha)^3} e^{-4\alpha\zeta} - \frac{S^2Pr^2A^2}{164\alpha^3(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} e^{-4SPr\zeta} - \frac{A^2}{32(SPr-\alpha)^3} e^{-4\alpha\zeta} \\
& + \frac{2SPrA}{4\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{A}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} \qquad (\text{when } PrS \neq a)
\end{aligned} \tag{36}
$$

In the limiting case, when $p \to 1$, Eq. (30) gives

$$
\begin{aligned}
\theta(\zeta) = {} & e^{-2SPr\zeta} + \frac{A}{4\alpha(SPr-\alpha)}(e^{-2\alpha\zeta} - e^{-2SPr\zeta}) \\
& + \Bigg[-\frac{A}{4(SPr-\alpha)^2} + \frac{SPrA}{4\alpha(SPr-\alpha)^2} + \epsilon\Bigg\{ 1 + \frac{A}{4(S^2Pr^2-\alpha^2)} \\
& - \frac{A}{2\alpha(SPr-\alpha)} + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} \\
& - \frac{A^2}{32\alpha(SPr-\alpha)^2(SPr-2\alpha)} - \frac{S^2Pr^2A^2}{16\alpha^3(SPr-\alpha)^2(SPr+\alpha)} \\
& - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} \\
& - \frac{A^2}{32(SPr-\alpha)^2(SPr-2\alpha)} + \frac{SPrA}{2\alpha(S^2Pr^2-\alpha^2)} \\
& - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} - \frac{A}{4\alpha(SPr-\alpha)} \Bigg\} \Bigg] e^{-2SPr\zeta} \\
& + \frac{\alpha A}{4\alpha(SPr-\alpha)^2} e^{-2\alpha\zeta} - \frac{SPrA}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} \\
& - \epsilon\Bigg\{ e^{-4SPr\zeta} + \frac{\alpha A}{4\alpha(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} - \frac{A}{2\alpha(SPr-\alpha)} e^{-4SPr\zeta} \\
& + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} - \frac{A^2}{32\alpha(SPr-\alpha)^3} e^{-4\alpha\zeta} \\
& - \frac{S^2Pr^2A^2}{16\alpha^3(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} e^{-4SPr\zeta} - \frac{A^2}{32(SPr-\alpha)^3} e^{-4\alpha\zeta} \\
& + \frac{2SPrA}{4\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \Bigg\} \\
& - \frac{A}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} + \cdots \qquad (\text{when } PrS \neq a)
\end{aligned} \tag{37}
$$

It is clear from Eq. (36) that the temperature distribution is characterized by four dimensionless parameters: the suction parameter S, the magnetic parameter M, the Prandtl number Pr, and the Brinkman number Br, where Br is a measure of viscous dissipation in the flow.

Using Fourier's law, the heat flux from the plate to the fluid can be written as

$$
q = -k(T)\frac{\mathrm{d}T}{\mathrm{d}z}\bigg|_{z=0} \tag{38}
$$

Using Eqs. (20), (22), and (36), heat flux in dimensionless form can be written as

$$
\begin{aligned}
q^* = {} & \frac{Q\sqrt{2\nu/\Omega}}{k_a(T_w - T_\infty)} = -(1+\epsilon\theta)\frac{\mathrm{d}\theta}{\mathrm{d}\zeta}\bigg|_{\zeta=0} \\
= {} & -\Bigg\{ 1 + \epsilon\Bigg[e^{-2SPr\zeta} + \frac{A}{4\alpha(SPrr-\alpha)}(e^{-2\alpha\zeta} - e^{-2SPrr\zeta}) \\
& + \Bigg[-\frac{A}{4(SPrr-\alpha)^2} + \frac{SPrA}{4\alpha(SPr-\alpha)^2} + \epsilon\Bigg\{ 1 + \frac{A}{4(S^2Pr^2-\alpha^2)} \\
& - \frac{A}{2\alpha(SPr-\alpha)} + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} - \frac{A^2}{32\alpha(SPr-\alpha)^2(SPr-2\alpha)} \\
& - \frac{S^2Pr^2A^2}{16\alpha^3(SPr-\alpha)^2(SPr+\alpha)} - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} \\
& + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} - \frac{A^2}{32(SPr-\alpha)^2(SPr-2\alpha)} + \frac{SPrA}{2\alpha(S^2Pr^2-\alpha^2)} \\
& - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} - \frac{A}{4\alpha(SPr-\alpha)} \Bigg\} \Bigg] e^{-2SPr\zeta} \\
& + \frac{\alpha A}{4\alpha(SPr-\alpha)^2} e^{-2\alpha\zeta} - \frac{SPrA}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} \\
& - \epsilon\Bigg\{ e^{-4SPr\zeta} + \frac{\alpha A}{4\alpha(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} - \frac{A}{2\alpha(SPr-\alpha)} e^{-4SPr\zeta} \\
& + \frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)} e^{-2\zeta(SPr+\alpha)} - \frac{A^2}{32\alpha(SPr-\alpha)^3} e^{-4\alpha\zeta} \\
& - \frac{S^2Pr^2A^2}{16\alpha^3(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& + \frac{A^2}{16\alpha^2(SPr-\alpha)^2} e^{-4SPr\zeta} - \frac{A^2}{32(SPr-\alpha)^3} e^{-4\alpha\zeta} \\
& + \frac{2SPrA}{4\alpha(SPr-\alpha)(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \\
& - \frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)} e^{-2\zeta(SPr+\alpha)} \Bigg\} \\
& - \frac{A}{4\alpha(SPr-\alpha)} e^{-2\alpha\zeta} + \cdots \Bigg] \Bigg\} \Bigg\{ -2SPr e^{-2SPr\zeta}
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
&+\frac{A}{4\alpha(SPr-\alpha)}(-2\alpha e^{-2\alpha\zeta}+2SPre^{-2SPr\zeta})\\
&-2SPr\Bigg[-\frac{A}{4(SPr-\alpha)^2}+\frac{SPrA}{4\alpha(SPr-\alpha)^2}\\
&+\epsilon\Bigg\{1+\frac{A}{4(S^2Pr^2-\alpha^2)}-\frac{A}{2\alpha(SPr-\alpha)}+\frac{S^2Pr^2A}{4\alpha^2(S^2Pr^2-\alpha^2)}\\
&-\frac{A^2}{32\alpha(SPr-\alpha)^2(SPr-2\alpha)}-\frac{S^2Pr^2A^2}{16\alpha^3(SPr-\alpha)^2(SPr+\alpha)}\\
&-\frac{A^2}{16\alpha(SPr-\alpha)(SPr+\alpha)}+\frac{A^2}{16\alpha^2(SPr-\alpha)^2}\\
&-\frac{A^2}{32(SPr-\alpha)^2(SPr-2\alpha)}+\frac{SPrA}{2\alpha(S^2Pr^2-\alpha^2)}\\
&-\frac{SPrA^2}{8\alpha^2(SPr-\alpha)^2(SPr+\alpha)}-\frac{A}{4\alpha(SPr-\alpha)}\Bigg\}\Bigg]e^{-2SPr\zeta}\\
&-\frac{\alpha A}{2(SPr-\alpha)^2}e^{-2\alpha\zeta}+\frac{SPrA}{2(SPr-\alpha)}e^{-2\alpha\zeta}\\
&-\epsilon\Bigg\{e^{-4SPr\zeta}-\frac{A}{2(SPr-\alpha)}e^{-2\zeta(SPr+\alpha)}+\frac{2SPrA}{\alpha(SPr-\alpha)}e^{-4SPr\zeta}\\
&-\frac{S^2Pr^2A}{2\alpha^2(SPr-\alpha)}e^{-2\zeta(SPr+\alpha)}+\frac{A^2}{8(SPr-\alpha)^3}e^{-4\alpha\zeta}\\
&+\frac{S^2Pr^2A^2}{8\alpha^3(SPr-\alpha)^2}e^{-2\zeta(SPr+\alpha)}-\frac{A^2}{8\alpha(SPr-\alpha)}e^{-2\zeta(SPr+\alpha)}\\
&-\frac{A^2}{4\alpha^2(SPr-\alpha)^2}e^{-4SPr\zeta}+\frac{\alpha A^2}{8(SPr-\alpha)^3}e^{-4\alpha\zeta}\\
&-\frac{SPrA}{\alpha(SPr-\alpha)}e^{-2\zeta(SPr+\alpha)}+\frac{SPrA^2}{4\alpha^2(SPr-\alpha)^2}e^{-2\zeta(SPr+\alpha)}\Bigg\}\\
&-\frac{A}{2(SPr-\alpha)}e^{-2\alpha\zeta}\Bigg\}
\end{aligned}
$$

where q^* is the dimensionless heat flux and, like temperature distribution, it also depends on the same four parameters.

VI. Results and Discussion

The effects of temperature-dependent thermal conductivity and Brinkman number Br on heat transfer for fixed values of S, Pr, and M are shown in Fig. 2. As expected, heat transfer is maximum at the surface and decreases with the increasing vertical distance ζ. It is clear from Fig. 2 that heat transfer from the disk decreases as the thermal conductivity decreases with temperature. In general, thermal conductivity decreases with the increase in temperature for most of the materials, and viscous dissipation reduces the dimensionless heat transfer, because viscous heating increases the fluid temperature range within the region $0 \le z \le \infty$, leading to higher differences between the disk and bulk temperatures. Exceptions to this behavior occur whenever there are suction and magnetic effects. Because of these effects, heat transfer at the surface increases with the increase in Brinkman number and decreases with the decrease in thermal conductivity.

Figure 3 shows the effects of variable thermal conductivity and magnetic parameter on heat transfer for fixed values of S, Pr, and Br. It was shown by Chakraborti et al. [21] that, in the presence of suction at the plate, the velocity component u increases with an increase in M, which increases the heat transfer at the surface in each case. The effect of viscous dissipation also helps in increasing the heat transfer from the plate. It is obvious from Fig. 3 that the heat transfer decreases with the vertical distance from the plate, and with the thermal conductivity as well. For fixed values of M, Br, and Pr, the effect of thermal conductivity on heat transfer is shown in Fig. 4. In this case, the suction parameter is varied, and the Brinkman number is kept low. It is obvious that the heat transfer increases with the increase in suction parameter. This is because of the increase of velocity due to suction. Again, the heat transfer decreases with the decrease in thermal conductivity, and this is due to the increase in temperature. The effect of thermal conductivity on heat transfer is

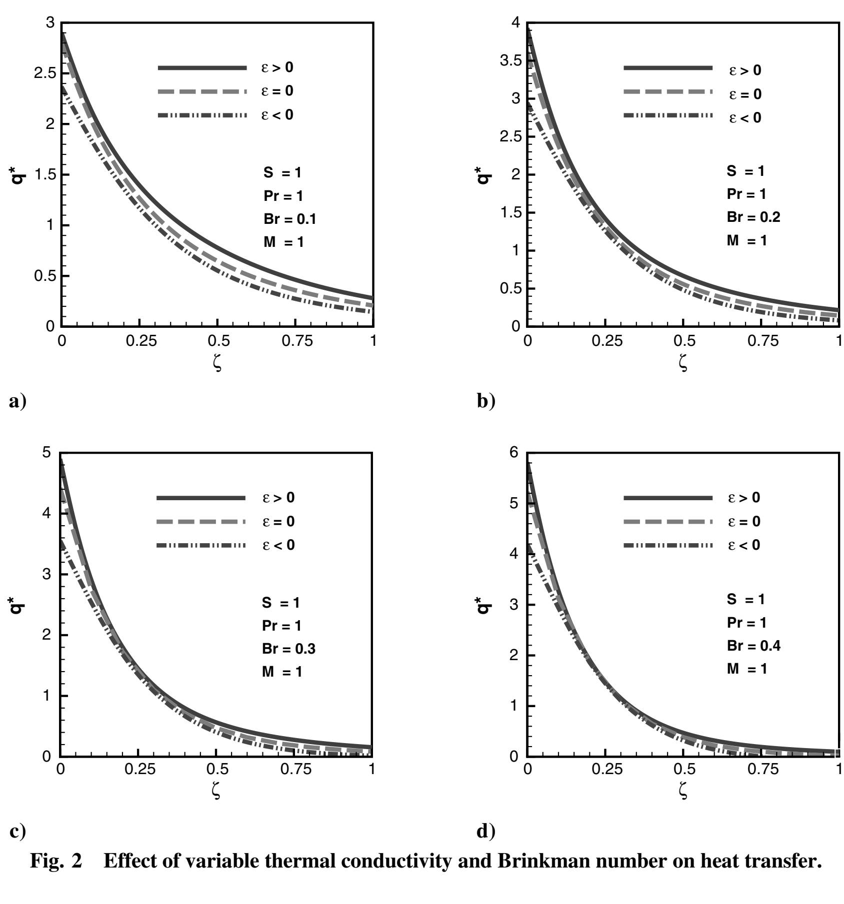

Fig. 2 Effect of variable thermal conductivity and Brinkman number on heat transfer.

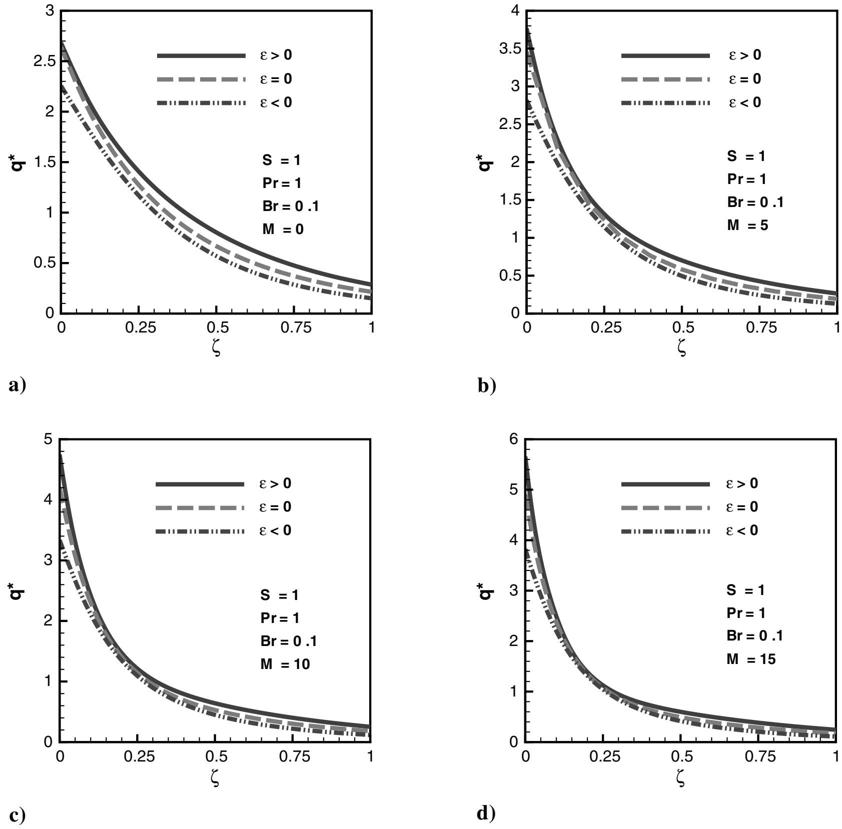

Fig. 3 Effect of variable thermal conductivity and magnetic parameter on heat transfer.

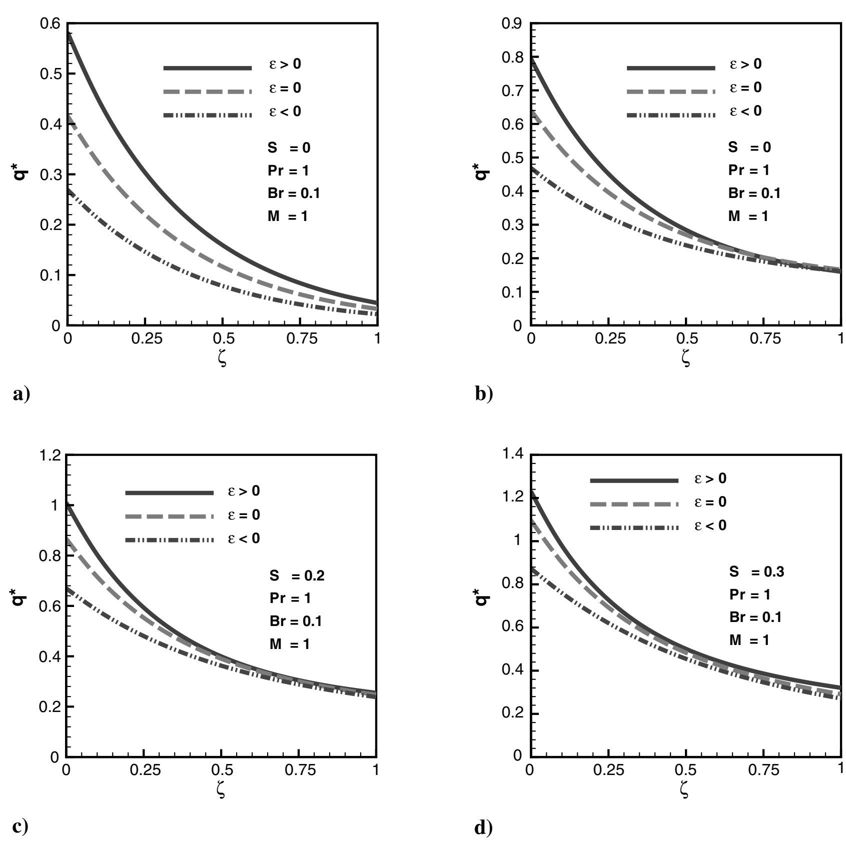

Fig. 4 Effect of variable thermal conductivity and suction on heat transfer.

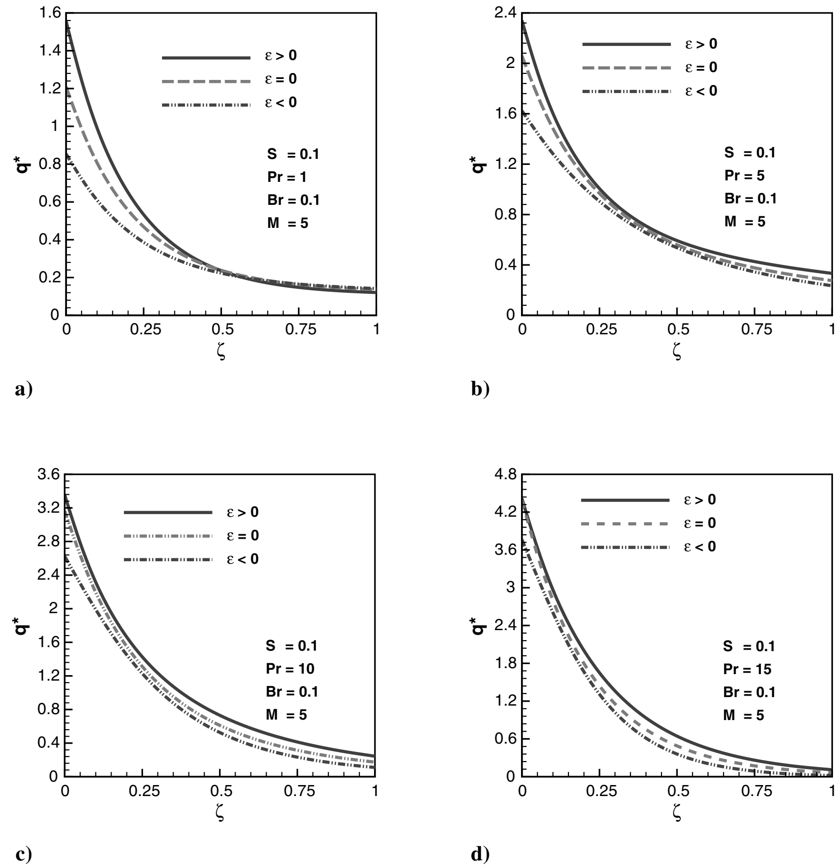

Fig. 5 Effect of variable thermal conductivity and Prandtl number on heat transfer.

more pronounced in the case of variable suction. The effects of suction with variable thermal conductivity on heat transfer from a porous disk are shown in Fig. 4 for a low Brinkman number. The heat transfer at the surface is maximum and decreases with the increase in vertical dimensionless distance in all cases. This heat transfer depends on the thermal conductivity of the disk and the suction parameter. Generally, thermal conductivity decreases with the increase in temperature, which decreases the heat transfer. This is obvious from Fig. 4 in all cases. Also, heat transfer increases with the increase in suction parameter. This effect is much more pronounced for higher Brinkman numbers. The heat transfer from the porous plate also depends on the Prandtl number. This effect is shown in Fig. 5 for the low-suction parameter. Again, the heat transfer at the surface is maximum and decreases as ζ increases. The effects of variable thermal conductivity are also the same as in previous cases. It is clear that heat transfer increases with the increase in the Prandtl numbers in all cases. This is due to the fact that increasing Pr decreases the thermal boundary-layer thickness for all S and Br values.

VII. Conclusions

The effects of temperature-dependent thermal conductivity on heat transfer are investigated for a hydromagnetic flow of an incompressible viscous electrically conducting fluid past a rotating porous plate. HPM is employed to solve a nonlinear energy equation, and the analysis is performed for different Br, S, M, and Pr numbers. It is demonstrated that the heat transfer from the surface of the porous plate 1) decreases with the decrease in thermal conductivity and 2) increases with the increase in Br, S, M, and Pr numbers.

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